In-use determination of the dynamical thermal properties of multilayer objects

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Abstract. Thermal response of multilayer objects to an arbitrary variation of temperatures of the ambient air are considered in terms of the step-response solutions (STS's). A procedure of calculation STS's with use of fast Fourier transform is described. A method of determination of STS's of multilayer objects in working conditions in the case of arbitrary varying temperatures of the ambient air is considered. An application of the developed technique to calculation of STS's of a model 3-layer object is demonstrated.

1. Introduction

One of the important tasks of nondestructive testing (NDT) in building construction and material engineering is a determination of thermal characteristics of materials of objects under investigation. A wide class of investigated objects has a multilayer structure, for example, in building construction all types of walls of buildings can be considered as multilayer objects. Thermal NDT of this kind of objects leads to the solution of the inverse problem of nonstationary heat conductivity, that is the determination of thermal characteristics (specific heat, density and thermal conductivity) of materials of layers constituting the object by means of examination of thermal response of the object.

A variety of methods allowing determination of heat-transfer properties of an object concerns the thermal response of the object to the ambient temperature variation of a specified type. For example, a determination of a (steady-state) thermal conductance (see Ref.[1]) of an object requires constant temperatures of the ambient air, while determination of the transient thermal characteristics (such as step-response functions (STS's)) requires stepped variation of an ambient air temperature (see Ref.[2,3]).

Nevertheless, frequently a thermal NDT deals with situations when one can not provide a specific regime of heat conductivity in object as, for example, in the case of thermal NDT of walls of a building, when one cannot control a temperature of the outdoor air. In this case the plausibility functional approach (see Ref.[4,5]) was proved to provide satisfactory results in a wide range of time dependencies of ambient air temperatures. The thermal characteristics of the layers are determined through the minimization of the functional, which can be performed only numerically. In practice, the only one characteristic is determined with numerical minimization procedure (as a rule, this is the thermal conductivity of the insulator layer, which mainly affects the thermal properties of the whole wall). All other characteristics are set equal corresponding design values.

Therefore, the approach loses the efficiency in cases when a large number characteristics are unknown due to substantial increasing numerical efforts.

On the other hand, thermal response of multilayer objects to an arbitrary variation of temperatures of the ambient air can be considered in terms of three STS's. These solutions are heat fluxes through object surfaces with temperatures of air near the object's surfaces varying as a step-function of time. STS's of the object provide information about heat-transfer coefficients of the surfaces of the object, its thermal conductance and characteristic period of thermal inertia of the object. Moreover, it is in principle possible to determine the structure of the object analyzing its STS's. In-use investigation of the object does not allow direct determination of the STS's because it is impossible to provide stepped variation of the ambient temperature in time. This situation takes place in NDT of buildings.

In the paper we present an approach to determination of STS's of multilayer objects in the case of arbitrary varying temperatures of the outside ambient. The approach is based on the minimization procedure analogous to that in plausibility functional approach. However, in the new approach the minimization is accomplished analytically, thus avoiding increasing numerical efforts.

The paper is organized as follows. In Sect. 2 we present a review of the approach to thermal response calculation with use of STS's. In Sect. 3 we describe the calculation of STS's by means of fast Fourier transform (FFT). In Sect. 4 we consider a determination of STS's of multilayer objects in working conditions in the case of arbitrary varying temperatures of the ambient air. In Sect.5 an application of the developed technique to calculation of STS's of a model 3-layer object is demonstrated. The paper ends with Sect.8 where main features of the approach are discussed.

2. Step-response solutions

Consider a one-dimensional heat conduction problem in a multilayered object of thickness L, when the heat conductivity λ, specific heat c and density ρ are piecewise-constant functions of coordinate z:

$$\rho(z)c(z)\frac{\partial T(z,t)}{\partial t} = \lambda(z)\frac{\partial^2 T(z,t)}{\partial^2 z}, \tag{1}$$

with boundary conditions

$$\left.\frac{\partial T(z,t)}{\partial z}\right|_{z=0} = \alpha_1\left(T(0,t) - T^{(1)}(t)\right), \quad \left.\frac{\partial T(z,t)}{\partial z}\right|_{z=L} = \alpha_2\left(T^{(2)}(t) - T(0,t)\right), \tag{2}$$

where α_i are heat-transfer coefficients of the left ($i=1$) and right ($i=2$) surfaces of the object (located at points $z=0$ and $z=L$, respectively), and $T^{(i)}(t)$ - are temperatures of the ambient air near the corresponding surfaces.

Let the temperature of air near the left boundary vary as a step function of time:

$$T^{(1)}(t) = \Theta(t), \; T^{(2)}(t) = 0, \tag{3}$$

where $\Theta(t)$ is a unit Heaviside step function. We will denote the corresponding heat flux through the left surface as $Q^{(11)}(t)$ and through the right surface as $Q^{(21)}(t)$. If we consider the step variation of the temperature of air near the right surface,

$$T^{(1)}(t) = 0, \; T^{(2)}(t) = \Theta(t), \tag{4}$$

we will calculate another heat flux through the right surface $Q^{(22)}(t)$. The heat flux through the left surface $Q^{(12)}(t)$ is equal in magnitude to $Q^{(21)}(t)$ but has an opposite sign, $Q^{(12)}(t) = -Q^{(21)}(t)$ (see Ref.[2]). Functions $Q^{(11)}(t)$, $Q^{(21)}(t)$ and $Q^{(22)}(t)$ are called STS's.

They provide information about heat-transfer coefficients, $\alpha_1 = Q^{(11)}(0)$ $\alpha_2 = -Q^{(22)}(0)$, and the thermal conductance K, $K = Q^{(11)}(\infty) = Q^{(21)}(\infty) = -Q^{(22)}(\infty)$.

Thermal response of a multilayer object to an arbitrary variation of temperatures $T^{(1)}(t)$ and $T^{(2)}(t)$, namely the heat fluxes through the left and right surfaces,: $I^{(1)}(t)$ and $I^{(2)}(t)$, respectively, can be expressed through the STS's:

$$I^{(1)}(t) = \int_0^\infty Q^{(11)}(\tau) \frac{\partial T^{(1)}(t-\tau)}{\partial t} d\tau - \int_0^\infty Q^{(21)}(\tau) \frac{\partial T^{(2)}(t-\tau)}{\partial t} d\tau \ , \tag{5}$$

$$I^{(2)}(t) = \int_0^\infty Q^{(21)}(\tau) \frac{\partial T^{(1)}(t-\tau)}{\partial t} d\tau + \int_0^\infty Q^{(22)}(\tau) \frac{\partial T^{(2)}(t-\tau)}{\partial t} d\tau \ . \tag{6}$$

Moreover, it is in principle possible to determine the structure of the object analyzing its STS's.

3. Calculation of STS's of multilayer object with use of FFT

As a rule, direct measurements of temperatures of ambient air with use of industrial sensors provide values of temperature at consecutive moments of time. Thus, we have to replace the continuous operation of integration in Eqs. (5) and (6) by an operation with discrete set of values of $T^{(1)}$ and $T^{(2)}$.

Consider the temperature history $T^{(1)}(t)$ and $T^{(2)}(t)$ measured at moments of time $t_j = \Delta t \cdot (j-1)$, with $j = 1...N$, where N is a total number of measurements and Δt is a time interval between consecutive measurements. We will use a notation $T_j^{(\alpha)} = T^{(\alpha)}(t_j)$, where $\alpha = 1,2$.

In order to calculate heat fluxes through the surfaces of the object consider $T_j^{(\alpha)}$ as a column-vector $\mathbf{T}^{(\alpha)}$ and apply FFT to $\mathbf{T}^{(\alpha)}$:

$$\mathbf{X}^{(\alpha)} = \mathrm{F} \times \mathbf{T}^{(\alpha)}, \quad \mathrm{F}_{nm} = \frac{1}{N} \exp\left(\frac{2\pi i}{N} (n-1)(m-1) \right), \tag{7}$$

where symbol $\times$ stands for matrix multiplication.

In order to interpolate the temperature histories $T^{(1)}(t)$ and $T^{(2)}(t)$ at whole time line we will use the corresponding Fourier series:

$$T^{(\alpha)}(t) = \sum_{k=1}^{N} X_k^{(\alpha)} \exp\left(-i\omega_k (t - t_1) \right), \tag{8}$$

where $\omega_k = \dfrac{2\pi}{\Delta t} \dfrac{k-1}{N+1}$ for $k \le \dfrac{N}{2}$, and $\omega_k = -\omega_{N-k+2}$ for $k > \dfrac{N}{2}$. The use of interpolating function (8) is quite reasonable because the Fourier series is a smooth function and it satisfies the condition: $T_j^{(\alpha)} = T^{(\alpha)}(t_j)$. Furthermore, the expansion (8) allows us to utilize the power of Fourier technique in analysing thermal response of the object.

It follows from Eq. (8) that the heat fluxes through the left and right surfaces at moment t_j, $I_j^{(1)}$ and $I_j^{(2)}$ respectively, are given by:

$$\mathbf{I}^{(\alpha)} = \sum_{\beta=1,2} \bar{\mathrm{F}} \times \left[\boldsymbol{\chi}^{(\alpha\beta)} \otimes \mathbf{X}^{(\beta)} \right], \quad \bar{\mathrm{F}}_{nm} = \exp\left(-\frac{2\pi i}{N} (n-1)(m-1) \right), \tag{9}$$

where symbol $\otimes$ stands for direct (elementwise) multiplication of two vectors. In Eq. (9) component $\chi_k^{(\alpha\beta)}$ of the vector $\boldsymbol{\chi}^{(\alpha\beta)}$ is an (complex) amplitude of the heat flux through the surface α ($\alpha = 1$ for the left surface and $\alpha = 2$ for the right surface) in the case when $T^{(\beta)}(t) = \exp\left(-i\omega_k(t - t_1)\right)$ and the temperature of air near the opposite surface is equal zero. For brevity, here we will omit the details of calculation of $\chi_k^{(\alpha\beta)}$, which can be found in Ref[7].

Making use of Eqs.(6)-(8) we can readily calculate STS's. First, we have to chose the time interval Δt. As a rule, Δt is chosen equal time interval between the consecutive measurements of the temperature histories which have to be analyzed with use of the STS's. Second, we have to choose the period of time $\bar{T}$ within which the STS's have to be calculated. As a rule, the period of time should be few times greater than the period of thermal inertia of the object τ_{in}.

In order to calculate, for example, $Q^{(11)}(t)$, we have to define the time moments $t_j = (j-1)\Delta t$, where $j = 1...2N$ with $N = \bar{T}/\Delta t$, and the temperature histories:

$$T_j^{(1)} = \begin{cases} 1, & j \le N \\ 0, & N < j \le 2N \end{cases}, \tag{10}$$

$$T_j^{(2)} = 0, \quad j = 1...2N. \tag{11}$$

Substituting Eqs. (10) and (11) into Eqs. (7)-(9) we can calculate vectors $\mathbf{I}^{(1)}$. Obviously the STS $Q^{(11)}(t)$ is defined by the elements of $\mathbf{I}^{(1)}$:

$$Q^{(11)}(t_j) = I_j^{(1)}, \qquad j = 1...N. \tag{12}$$

STS's $Q^{(21)}(t)$ and $Q^{(22)}(t)$ are calculated in completely analogous manner.

Consider situation when we know the STS's $Q^{(11)}(t_j)$, $Q^{(21)}(t_j)$ and $Q^{(22)}(t_j)$, and the temperature histories $T_j^{(1)}$ and $T_j^{(2)}$, where $j = 1...N$, and we need to calculate the heat fluxes through the surfaces of the object. First, we will consider the temperature histories $T_j^{(\alpha)}$ as a periodic sequence with period N. Let us construct a sequence with period $6N$:

$$\tilde{T}_j^{(\alpha)} = \sum_{k=1}^{N} \left[H_{j-k} + H_{j-N-k} + H_{N+j-k}\right]\left[T_k - T_{k-1}\right], \tag{13}$$

where H_j is a periodic sequence with period $6N$:

$$H_j = \begin{cases} 1, & j = 1...3N \\ 0, & j = 3N+1...6N \end{cases}. \tag{14}$$

Obviously, $\tilde{T}_j^{(\alpha)} = T_j^{(\alpha)}$ for $j = 1-N...2N$, therefore, the heat fluxes at moments $t_j = (j-1)\Delta t$, where $j = 1...N$, are given by

$$I_j^{(\alpha)} = \sum_{\beta=1,2}\sum_{k=1}^{N} \left[Q_{j-k}^{(\alpha\beta)} + Q_{j-N-k}^{(\alpha\beta)} + Q_{j+N-k}^{(\alpha\beta)}\right]\left[T_k^{(\alpha)} - T_{k-1}^{(\alpha)}\right], \tag{15}$$

where $Q_j^{(\alpha\beta)}$ is a periodic sequence (period $6N$):

$$Q_j^{(\alpha\beta)} = \begin{cases} Q^{(\alpha\beta)}(t_j), & j = 1...3N \\ Q_j^{(\alpha\beta)}, & j = 3N+1...6N \end{cases}. \tag{16}$$

4. Determination of STS's in the case of arbitrary temperature histories

The situation which frequently takes place in the practice of NDT is that multilayer object under investigation can not be placed inside the climatic chamber and subjected to specifically varying temperatures of ambient air. In this section we consider a method of determination of STS' of multilayer object in the case of arbitrary varying temperatures of ambient air.

Suppose we know a temperature history of ambient air $T_j^{(\alpha)}$, $\alpha = 1, 2$, $j = 1 \ldots N$, and heat fluxes through the surfaces of the object $I_j^{(\alpha)}$. The total duration of measurements $\tau_m = \Delta t(N-1)$ should be few times greater than the period of the thermal inertia of the object τ_{in}. Let us also chose the time moment t_{j*} which is about two times greater than τ_{in}. The values of heat fluxes after t_{j*} almost do not depend on the initial distribution of the temperature inside the object.

In order to determine STS's let us construct approximate STS's $\tilde{Q}_j^{(\alpha\beta)}$ which are linear combination of some (defined) sequences $f_{k,j}^{(\alpha\beta)}$:

$$\tilde{Q}_j^{(\alpha\beta)} = \begin{cases} \sum\limits_{k=1}^{M} \beta_k^{(\alpha\beta)} f_{k,j}^{(\alpha\beta)} + K\left[\Theta(t_j - \bar{t}) + \Theta(\bar{t} - t_j + 1) f_{M+1,j}^{(\alpha\beta)}\right], & \beta = 1 \\ \sum\limits_{k=1}^{M} \beta_k^{(\alpha\beta)} f_{k,j}^{(\alpha\beta)} - K\left[\Theta(t_j - \bar{t}) + \Theta(\bar{t} - t_j + 1) f_{M+1,j}^{(\alpha\beta)}\right], & \beta = 2 \end{cases}, \tag{17}$$

where $\beta_k^{(\alpha\beta)}$ are some (unknown) coefficients, and K is (unknown) thermal conductance of the object. There are in general no limitations on the choice of sequences $f_{k,j}^{(\alpha\beta)}$. An example of choice of $f_{k,j}^{(\alpha\beta)}$ is demonstrated in Sect.5.

Our goal is to choose the values of $\beta_k^{(\alpha\beta)}$ to minimize the discrepancy between the true STS's $Q_j^{(\alpha\beta)}$ and the approximate sequences $\tilde{Q}_j^{(\alpha\beta)}$. In order to do this let us calculate the heat fluxes $\tilde{I}_{j,k}^{(\alpha\beta)}$ corresponding to the basic sequences $f_{k,j}^{(\alpha\beta)}$. Procedure of calculation of $\tilde{I}_{j,k}^{(\alpha\beta)}$ is described in Sect.3(see Eqs. (13)-(16)).

Let us introduce three $(N-j*)\times M$ matrices $\mathrm{I}^{(11)}$, $\mathrm{I}^{(21)}$ and $\mathrm{I}^{(22)}$ with elements:

$$I_{j,k}^{(\alpha\beta)} = \tilde{I}_{j+j*,k}^{(\alpha\beta)}, \quad j = 1 \ldots (N-j*), \tag{18}$$

a (column) vector $\boldsymbol{\beta}$ of length $3M+1$ with elements:

$$\beta_j = \begin{cases} \beta_j^{(11)}, & j = 1 \ldots M \\ \beta_{j-M}^{(21)}, & j = M+1 \ldots 2M \\ \beta_{j-2M}^{(22)}, & j = 2M+1 \ldots 3M \\ K, & j = 3M+1 \end{cases}, \tag{19}$$

a (column) vector $\boldsymbol{\delta}\mathbf{T}$ of length $(N-j*)$ with elements:

$$\delta T_j = T_{j+j*}^{(1)} - T_{j+j*}^{(2)}, \quad j = 1 \ldots (N-j*) \tag{20}$$

and a (column) vector $\bar{\mathbf{I}}$ of length $2(N-j*)$ with elements:

$$\bar{I}_j = \begin{cases} I_{j+j*}^{(1)}, & j = 1 \ldots (N-j*) \\ I_{j+2j*-N}^{(2)}, & j = (N-j*)+1 \ldots 2(N-j*) \end{cases}. \tag{21}$$

If the approximated sequence (17) gives the exact values for true STS's than the vector $\bar{\mathbf{I}}$ has to coincide with the vector $\bar{\mathbf{I}}^{(c)}$ defined by

$$\bar{\mathbf{I}}^{(c)} = \Lambda \times \boldsymbol{\beta}, \tag{22}$$

where Λ is a $2(N-j*)\times 3M$ block-matrix:

$$\Lambda = \begin{pmatrix} \mathrm{I}^{(11)} & -\mathrm{I}^{(21)} & 0 & \boldsymbol{\delta}\mathbf{T} \\ 0 & \mathrm{I}^{(21)} & \mathrm{I}^{(22)} & -\boldsymbol{\delta}\mathbf{T} \end{pmatrix}. \tag{23}$$

In practice due to errors of approximation (17) and errors of measurements of temperatures and heat fluxes we can only require a minimal value of the discrepancy:

$$D(\boldsymbol{\beta}) = \left| \overline{\mathbf{I}}^{(c)}(\boldsymbol{\beta}) - \overline{\mathbf{I}} \right|^2 \qquad (24).$$

Thus, the unknown parameters $\boldsymbol{\beta}$ are given by:

$$\boldsymbol{\beta} = \left[\Lambda^{tr}\Lambda\right]^{-1} \Lambda^{tr}\overline{\mathbf{I}}, \tag{25}$$

where the superscript tr denotes the operation of transpose.

5. Example of application of the developed formalism

In this section we demonstrate the application of the developed technique to determination of STS's of a model 3-layer object. Thermal characteristics of materials constituting the object a listed in Tab. 1.

Layer material	Heat conductivity Wt/(m °C)	Density kg/m^3	Specific heat J/(kg °C)	Thickness mm
Reinforced concrete	2.04	2400	840	70
Insulator	0.4	70	1470	100
Ceramic tile	1.28	2800	840	30

Table 1. Thermal characteristics of layers of the model 3-layer object.

We have simulated the temperature histories of the ambient air: the time interval between the consecutive measurements was set equal $\Delta t = 300$ sec., the total duration of measurements $\overline{T} = 10$ days and the period of time for the calculation of the discrepancies $[t_{j*}...\overline{T}]$, where $t_{j*} = 4$ days. The following formulas were used for simulation:

$$T^{(1)}(t) = 10 + \sum_{n=1}^{9} \sin\left(\frac{2\pi}{\tau_n^{(1)}} t\right), \tag{26}$$

$$T^{(2)}(t) = -10 + 2\sum_{n=1}^{9} \sin\left(\frac{2\pi}{\tau_n^{(2)}} t\right), \tag{27}$$

where $\tau_n^{(1)} = \{0.5, 0.8, 0.9, 1, 1.3, 1.5, 2, 3, 6\}$ days, and $\tau_n^{(2)} = \{0.5, 0.7, 0.8, 1, 1.2, 1.5, 2, 3, 6\}$ days. Heat fluxes corresponding to these temperature histories were calculated as described in Sect.3. The temperature histories and heat fluxes are presented at Fig. 1.

For the determination of STS's on the basis of simulated histories (see Fig. 1) with the technique described in Sect. 4. we have used the following sequences $f_{k,j}^{(\alpha\beta)}$, where $k = 1...6$:

$$f_{k,j}^{(\alpha\beta)} = \mathrm{F}\left(t_j, \mathbf{y}_k^{(\alpha\beta)}, \mathbf{x}^{(\alpha\beta)}\right), \tag{28}$$

where function F is a (cubic) spline interpolation of discrete function defined by set of values $\mathbf{y}_k^{(\alpha\beta)}$ calculated at points $\mathbf{x}^{(\alpha\beta)}$ to moments t_j.

We use the following values for components of $\mathbf{x}^{(\alpha\beta)}$ and $\mathbf{y}_k^{(\alpha\beta)}$:

$$\mathbf{x}^{(11)} = \mathbf{x}^{(22)} = \{0, 0.1, 0.2, 0.3, 0.6, 0.8, 1.2\}, \tag{29}$$

$$\mathbf{x}^{(21)} = \{0, 0.05, 0.1, 0.2, 0.4, 0.7, 0.9, 1.2\}, \tag{30}$$

$$y_{j,k}^{(11)} = y_{j,k}^{(22)} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}, \tag{31}$$

$$y_{j,k}^{(21)} = \begin{cases} 1, & j = k - 1 \\ 0, & j \neq k - 1 \end{cases}. \tag{32}$$

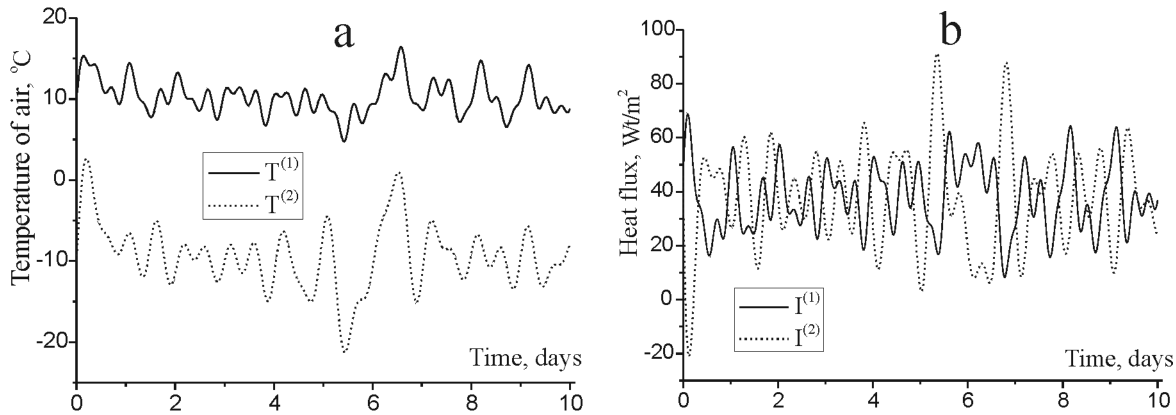

Figure 1. Temperature (**a**) and heat fluxes (**b**) simulated with use of Eqs. (26) and (27).

Thus, the unknown parameters $\beta_k^{(\alpha\beta)}$ are the set of values of the STS's calculated at points $\mathbf{x}^{(\alpha\beta)}$. These parameters were calculated with use of the technique described in Sect. 4. The true STS's for the object and the approximated sequences are plotted at the Fig. 2 (**a**).

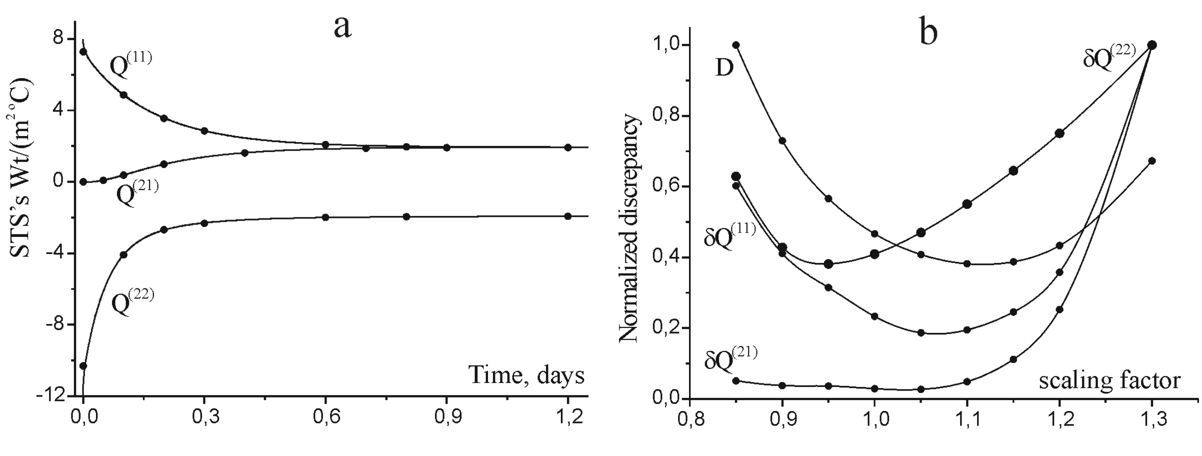

Figure 2. Panel **a**: exact STS's (solid curves) and determined values of the STS's (circles) calculated at points $\mathbf{x}^{(\alpha\beta)}$. Panel **b**: quadratic discrepancies $\delta Q^{(\alpha\beta)}$ between the exact STS's and the approximate expressions $\tilde{Q}_j^{(\alpha\beta)}$ and also the discrepancy $D(\tilde{\boldsymbol{\beta}})$, where $\tilde{\boldsymbol{\beta}}$ are the determined parameters, plotted as a functions of scaling factor.

Determined values of parameters $\boldsymbol{\beta}$ provides a satisfactory approximation of the STS's: the relative discrepancy between the exact STS's and the approximate sequences does not exceed 1 % for moments of time $t_j > 2.5$ hours. Within the time interval

$t_j < 2.5$ hours the relative discrepancy substantially increases and for the STS $\tilde{Q}^{(22)}(0)$ the discrepancy is about 7 %. The $Q^{(22)}(t)$ at small times t has a very large derivatives of all orders, therefore the (cubic) spline interpolation results in essential errors of approximation of $Q^{(22)}(t)$ in this time region. However, the error of interpolation can be reduced by increasing the number of points for interpolation at this region.

We have chosen the points for interpolation $\mathbf{x}^{(\alpha\beta)}$ to provide a precise spline interpolation for the STS's. The points of $\mathbf{x}^{(\alpha\beta)}$ are not uniformly distributed: they are concentrated, primarily, near the region with a large second derivative of STS's. The different multilayer object have similar STS's: the most essential difference is in the characteristic period of thermal inertia of objects. This means that the most appropriate set of points for interpolation $\overline{\mathbf{x}}^{(\alpha\beta)}$ for other objects differs from $\mathbf{x}^{(\alpha\beta)}$ by a scale factor γ :

$$\overline{\mathbf{x}}^{(\alpha\beta)} = \gamma\,\mathbf{x}^{(\alpha\beta)}. \tag{33}$$

We have performed the calculations of parameters $\boldsymbol{\beta}$ for different scale factors γ , namely, we have used the vectors $\overline{\mathbf{x}}^{(\alpha\beta)}$ instead $\mathbf{x}^{(\alpha\beta)}$ in Eqs. (29) and (30). The discrepancies for different values of scale factor γ are plotted in Fig.2 (**b**).

The position of the minimum of the discrepancy $D(\gamma)$ does not coincide with that of STS's. The reason is in errors of the spline interpolation. The situation can be improved by increasing the number M of points for interpolation. Nevertheless, as we have mentioned above, the discrepancy between the true STS's and the approximate sequences are sufficiently small even for $M = 6$.

6. Discussions and conclusions

Generally, the technique developed in the paper can be considered as an extension of the plausibility functional approach on the case when the internal structure and the thermal characteristics of the layers of the object is a priori unknown. The unknown parameters $\boldsymbol{\beta}$ are determined from the procedure of minimization of the quadratic discrepancy between measured heat fluxes and heat fluxes calculated on the basis of temperature histories. The essential difference is that in the developed technique we express the discrepancy through the parameters $\boldsymbol{\beta}$ analytically, so we can analytically minimize the discrepancy (see Eq. (25)).

In general, there are no limitations on the choice of $f_{k,j}^{(\alpha\beta)}$. In the example of application of the technique we have used the spline functions as the basis of expansion (17) because we think that this functions provides simple and universal approximation of STS's. The expansion results in errors at small times t_j, which can be reduced if we increase the number of the interpolation points $\mathbf{x}^{(\alpha\beta)}$. On the other hand, an increase of M requires increasing the period of measurements to provide enough information for determination of the parameters $\boldsymbol{\beta}$. In general, the number of the Fourier harmonics in temperature histories has to be greater than the number of determined parameters. In the example demonstrated in Sect.5 the increasing of the number of parameters $M = 8$ leads to physically meaningless results (STS's oscillated at small times).

There are two types of errors of determination of $\boldsymbol{\beta}$. First one is errors due to the initial distribution of the temperature inside the object. The initial distribution of the temperature affects the heat fluxes through the surfaces only at the time interval which is comparable with the period τ_{in} of thermal inertia of the object. Therefore, the corresponding errors can be estimated as

$$\Delta_{in}\boldsymbol{\beta} \approx \boldsymbol{\beta}\exp\left(-\tau_m/\tau_{in}\right). \qquad (34)$$

In our example we have used $\tau_m = 4$ days which are approximately 4 times greater than τ_{in}, therefore we could neglect this type of errors.

Second one is errors due to the random errors of measurements of temperatures and heat fluxes. This type of errors strongly affects the determined values of $Q^{(\alpha\beta)}(t)$ at small times t. The corresponding errors can be estimated as:

$$\Delta_r\boldsymbol{\beta} \approx \boldsymbol{\beta}\frac{\delta T}{<\Delta T>}, \qquad (35)$$

where, δT is an error of measurements and $<\Delta T>$ is a mean modulus of the difference between the consecutively measured temperatures. This type of errors can be reduced by averaging the measured data:

$$\Delta_r\boldsymbol{\beta} \propto \frac{1}{\sqrt{T_{av}/\Delta t}}, \qquad (36)$$

where T_{av} is an averaging period. It should be noted, that averaging results in loss of information about STS's at period of time $[0...T_{av}]$.

To summarize, we have developed technique for determination of the dynamical thermal properties of multilayer object. The thermal characteristics a determined with use of the procedure of minimization of the discrepancy between the measured heat fluxes and that calculated on the basis of the measured temperature histories. The technique does not require a priori data about the internal structure of the object and can be applied in a wide range of temperature histories. The typical error of determination of STS's is less than 1%.

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